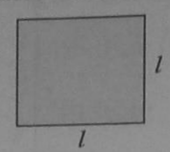
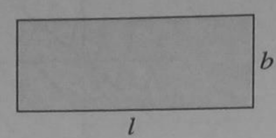
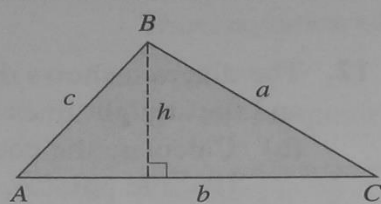
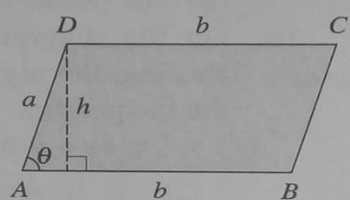
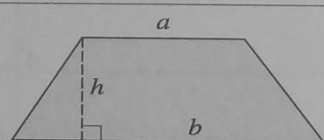
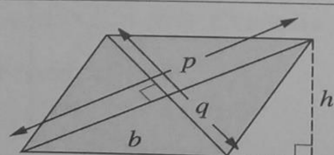
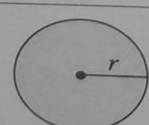
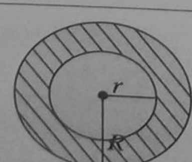
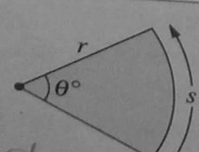
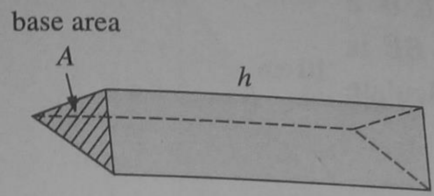
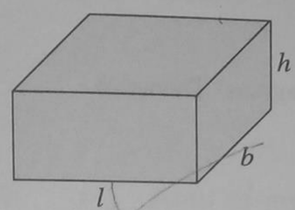
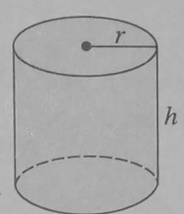
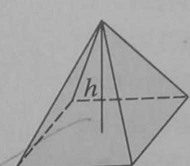
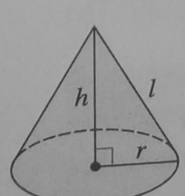
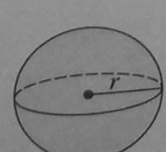
Mathematics Grade 10 & Grade 11

Figure	Diagram	Formulae
Square		Area $= l^2$ Perimeter $= 4l$
Rectangle		Area $= l \times b$ Perimeter $= 2(l + b)$
Triangle		Area $= \frac{1}{2}$base $\times$ height $= \frac{1}{2} b \times h$ $= \frac{1}{2} ab \sin C$
Parallelogram		Area = base $\times$ height $= b \times h$ $= ab \sin \theta$
Trapezium		Area $= \frac{1}{2}(a + b)h$
Rhombus		Area $= \frac{1}{2} p \times q$ $= b \times h$
Circle		Area $= \pi r^2$ Circumference $= 2\pi r$
Annulus		Area $= \pi(R^2 - r^2)$
Sector		Arc length $s = \frac{\theta°}{360°} \times 2\pi r$ Area $= \frac{\theta°}{360°} \times \pi r^2$ Perimeter $= \frac{\theta°}{360°} \times 2\pi r + 2r$

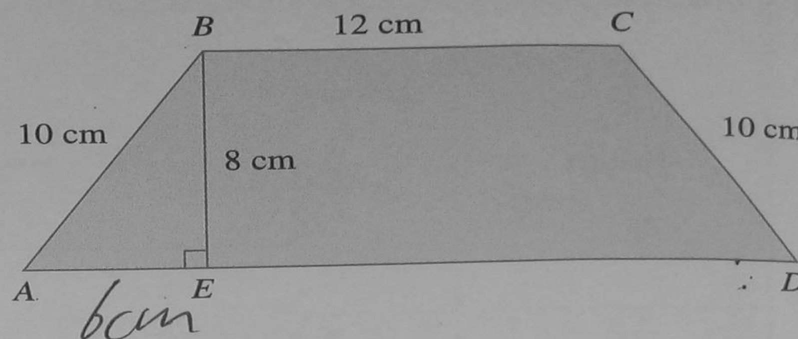
Solid	Diagram	Formulae
Prism		Volume = area of cross-section × length = base area × height $= Ah$ Total surface area = perimeter of the base × height + 2(base area)
Cuboid		Volume $= Ah$ $= l \times b \times h$ Total surface area $= 2(lb + lh + bh)$
Cylinder		Volume $= Ah$ $= \pi r^2 h$ Curved surface area $= 2\pi rh$ Total surface area $= 2\pi rh + 2\pi r^2$ (for solid cylinder or closed at both ends)
Pyramid		Volume $= \frac{1}{3}$base area $\times h$
Cone		Volume $= \frac{1}{3}$ base area $\times h$ $= \frac{1}{3}\pi r^2 h$ Curved surface area $= \pi rl$ (where l is the slant height) Total surface area $= \pi rl + \pi r^2$ (for solid cone)
Sphere		Volume $= \frac{4}{3}\pi r^3$ Surface area $= 4\pi r^2$

Example 34

$ABCD$ is a trapezium in which BC is parallel to AD. E is a point on AD such that BE is perpendicular to AD. Calculate

(a) AE,

(b) the area of $ABCD$.



Solution

(a) $AE^2 + 8^2 = 10^2$ (Pythagoras' Theorem)

$AE^2 + 64 = 100$

$AE^2 = 36$

$AE = 6$ cm

(b) $ABCD$ is a symmetrical trapezium

$\therefore \quad FD = AE = 6$ cm

$EF = BC = 12$ cm

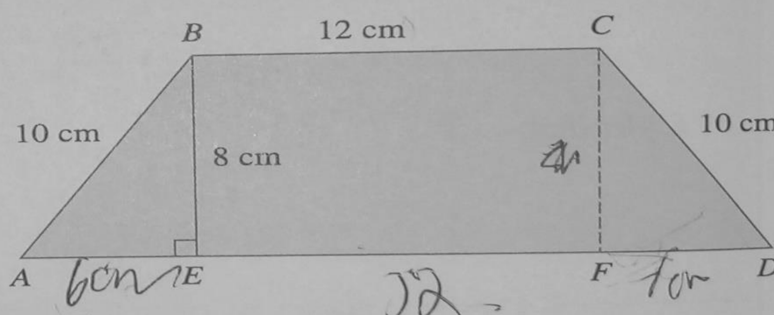
$\therefore \quad AD = 6 + 12 + 6$

$= 24$ cm

$\therefore$ area of trapezium

$ABCD = \frac{1}{2}(12 + 24) \times 8$

$= 144 \text{ cm}^2$



×8
12+
6
18+
17
36

30
×4
120

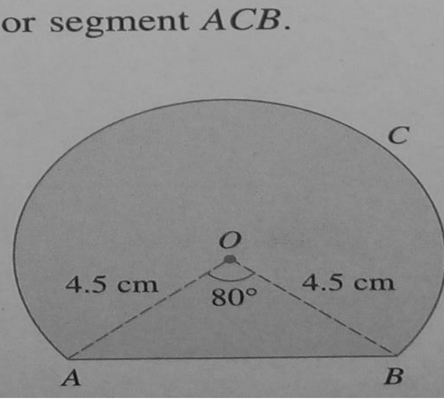
Example 35

The cross-section of a tunnel is the major segment ABC of a circle centre O as shown in the diagram. Given that $OA = OB = 4.5$ m, $A\hat{O}B = 80°$ and taking $\pi = \frac{22}{7}$, calculate

(a) the length of the arc ACB,

(b) the area of the triangle AOB,

(c) the area of the major segment ACB.



Solution

(a) Reflex $\angle AOB = 360° - 80°$ ($\angle$s at a pt.)
$= 280°$

$\therefore$ length of arc $ACB = \frac{280°}{360°} \times 2\pi r$
$= \frac{280}{360} \times 2 \times \frac{22}{7} \times 4.5$
$= 22$ m

(b) Area of $\triangle AOB = \frac{1}{2}ab \sin \theta$
$= \frac{1}{2}(4.5)(4.5) \sin 80°$
$= 9.97 \text{ m}^2$ (correct to 3 sig. fig.)

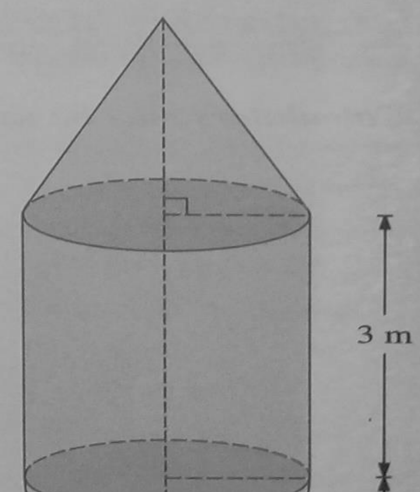
(c) Area of major sector $= \frac{280°}{360°} \times \pi r^2$
$= \frac{280}{360} \times \frac{22}{7} \times 4.5^2$
$= 49.5 \text{ m}^2$

$\therefore$ Area of major segment ACB = area of sector + area of $\triangle$
$= 49.5 + 9.97$
$= 59.47$
$= 59.5 \text{ m}^2$ (correct to 3 sig. fig.)

Example 36

A container for rocket fuel is made by joining together a cone, a cylinder and a hemisphere as shown. Taking $\pi = 3.142$, calculate

(a) the volume of the cylinder,
(b) the volume of the hemisphere,
(c) the height of the cone if the total volume of the container is 3 m^3,
(d) the total surface area of the container.



Solution

(a) Volume of cylinder $= \pi r^2 h$
$= 3.142 \times 0.5^2 \times 3$
$= 2.356\,5$
$= 2.36 \text{ m}^3$ (correct to 3 sig. fig.)

(b) Volume of hemisphere $= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$
$= \frac{2}{3} \times 3.142 \times 0.5^3$
$= 0.261\,8$
$= 0.262 \text{ m}^3$ (correct to 3 sig. fig.)

(c) Volume of cone = 3 – (2.356 5 + 0.261 8)

$$\frac{1}{3}\pi r^2 h = 0.381\ 7\ \text{m}^3$$

$$h = \frac{3 \times 0.381\ 7}{3.142 \times 0.5^2}$$

$$= 1.457\ 8$$

$$= 1.46\ \text{m}$$

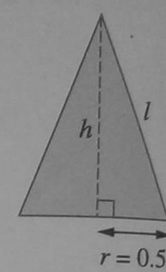
(d) Let the slant height of the cone be l cm.

$\therefore \quad l^2 = r^2 + h^2$ (Pythagoras' Theorem)

$= 0.5^2 + 1.457\ 8^2$

$= 2.375\ 2$

$l = 1.541$ m



Surface area of cone = πrl

$= 3.142 \times 0.5 \times 1.541 = 2.420\ 9\ \text{m}^2$

Curve surface area of cylinder = $2\pi rH$

$= 2 \times 3.142 \times 0.5 \times 3 = 9.426\ \text{m}^2$

Surface area of hemisphere = $\frac{1}{2}(4\pi r^2)$

$= 2 \times 3.142 \times 0.5^2 = 1.571\ \text{m}^2$

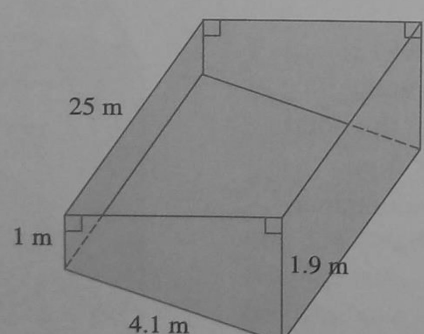
$\therefore$ total surface area = 2.4209 + 9.426 + 1.571

$= 13.4\ \text{m}^2$ (correct to 3 sig. fig.)

Exercise 10E

Calculators may be used only for questions marked with an asterisk (*).

1.



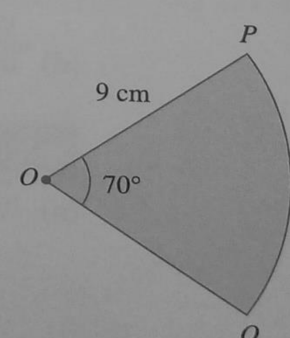
A 25-metre long swimming pool has a trapezoidal cross-section such that it is 1 m deep on one side and 1.9 m deep on the other side. The bottom slopes uniformly from one side to the other. The sloping length is 4.1 m. Calculate

(a) the breadth of the swimming pool,

(b) the volume of the swimming pool if it is full,

(c) the surface area of the inside of the swimming pool if it is empty.

2.

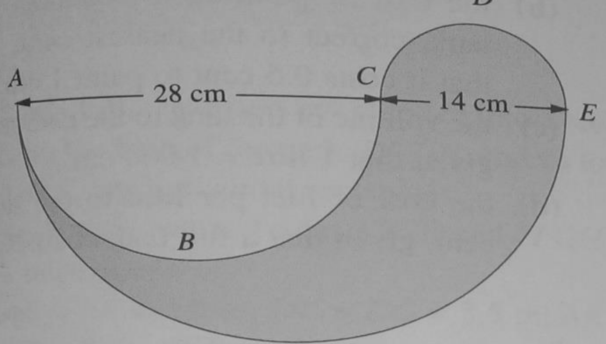


The diagram shows sector OPQ of a circle, centre O, radius 9 cm, in which $P\hat{O}Q = 70°$. Taking $\pi = \frac{22}{7}$, calculate

(a) the perimeter of the sector,

(b) the area of the sector.

3. A metal sheet is cut into the shape shown in the diagram. The perimeter of the sheet consists of three semicircular arcs ABC, CDE and EFA.

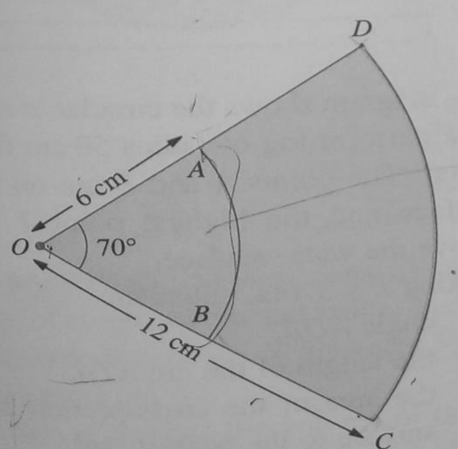


The diameters, $AC = 28$ cm and $CE = 14$ cm.

Taking $\pi = \frac{22}{7}$, calculate

(a) the length of the semicircular arc ABC,
(b) the perimeter of the sheet,
(c) the area of the sheet.

4.

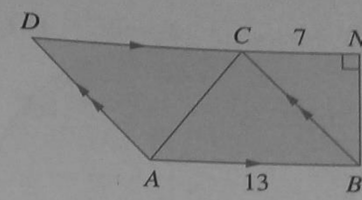


The diagram shows two arcs, AB and CD, of concentric circles, centre O. The radii OA and OC are 6 cm and 12 cm respectively, and $A\hat{O}B = 70°$.

Taking $\pi = \frac{22}{7}$, calculate

(a) the area of sector AOB,
(b) the area of the shaded region $ABCD$,
(c) the perimeter of the shaded region $ABCD$.

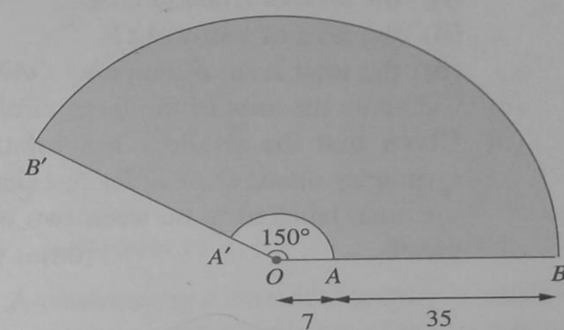
5.



$ABCD$ is a parallelogram.
BN is perpendicular to DC produced.
The area of triangle ABC is 39 cm^2, $AB = 13$ cm and $CN = 7$ cm.
Calculate

(a) the area of the parallelogram $ABCD$,
(b) the length of BN,
(c) the area of $ABNC$. [O/Jun 94/I]

*6.



In this question either take the value of π to be 3.142 or use the value on your calculator.

The coloured area in the diagram represents the part of the flat windscreen of a car which is being wiped by the windscreen wiper AB. The wiper rotates through 150° about O.
$OA = OA' = 7$ cm and $AB = A'B' = 35$ cm.
Calculate

(a) the length of the arc BB',
(b) the ratio of the arc lengths, $AA' : BB'$,
(c) the area of the screen which is wiped. [O/Nov 95/II]

*7. **In this question take π to be 3.142.**
The diagram shows a window in a large church.
AXB is an arc of circle C. The lines OA and OB are tangents to this circle. The other four panels are each identical to $OAXB$.
O is the centre of the large circle which touches arc AXB at X.

Workout Exercise: 1 to 6.